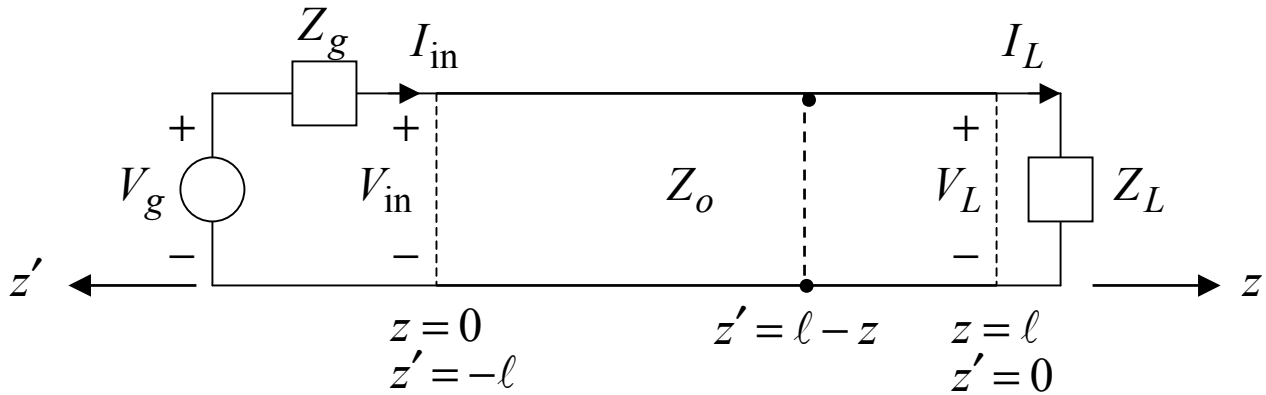
Exact voltage and current anywhere on a finite transmission line (D. K. Cheng)

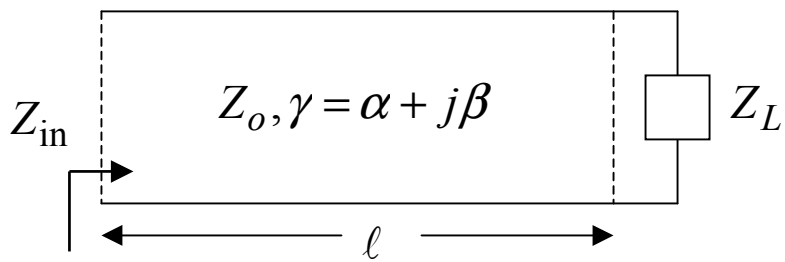


Expressions for the exact phasor voltage and current:

$$V(z') = \underbrace{\frac{Z_o V_g}{Z_o + Z_g} e^{-\gamma z}}_{V_{\text{in}}} \left(\frac{1 + \Gamma_L e^{-2\gamma z'}}{1 - \Gamma_g \Gamma_L e^{-2\gamma \ell}} \right)$$

$$I(z') = \frac{V_g}{Z_o + Z_g} e^{-\gamma z} \left(\frac{1 - \Gamma_L e^{-2\gamma z'}}{1 - \Gamma_g \Gamma_L e^{-2\gamma \ell}} \right)$$

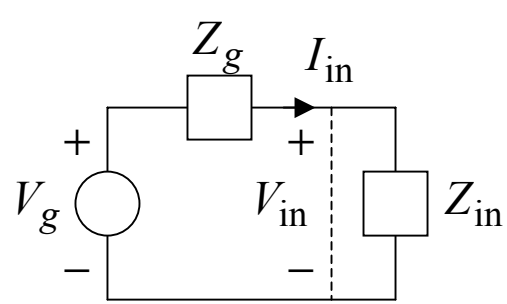
For a lossy line terminated with a load:



The input impedance is

$$Z_{\text{in}} = Z_o \left(\frac{Z_L + Z_o \tanh(\gamma \ell)}{Z_o + Z_L \tanh(\gamma \ell)} \right)$$

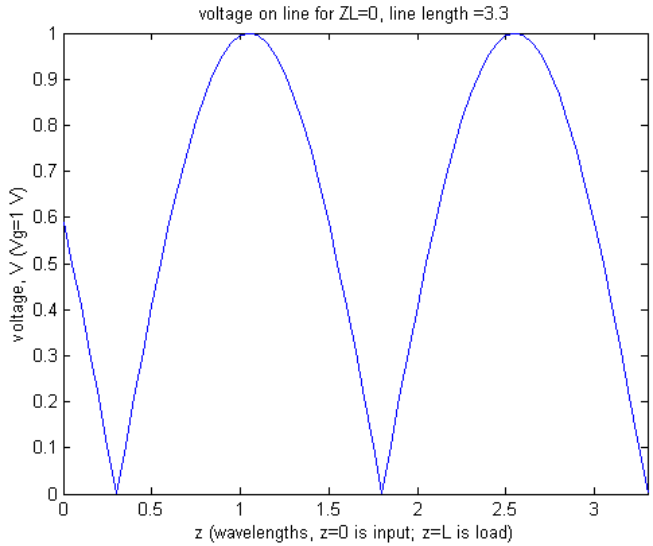
The equivalent circuit for computing the power into the line:

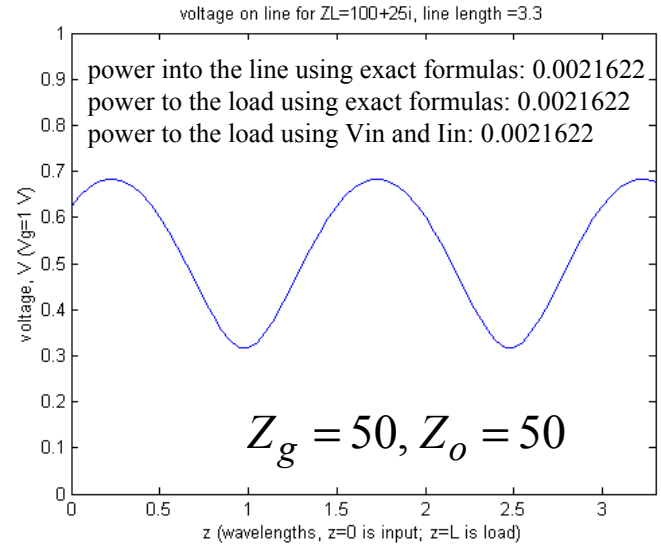


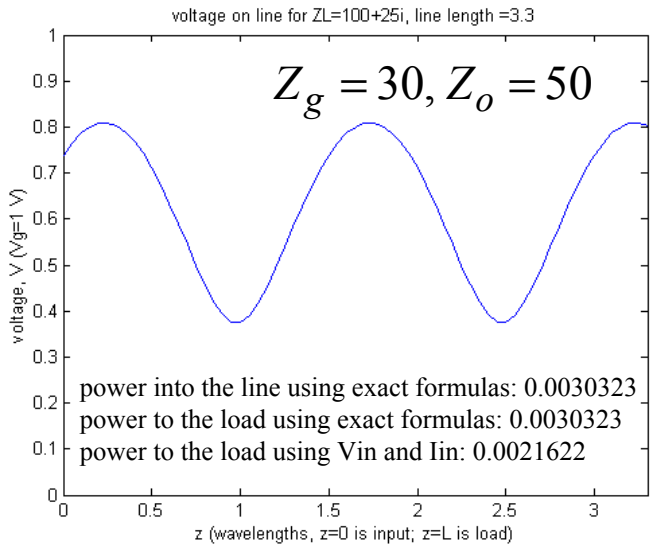
Matlab program
XlinePower.m

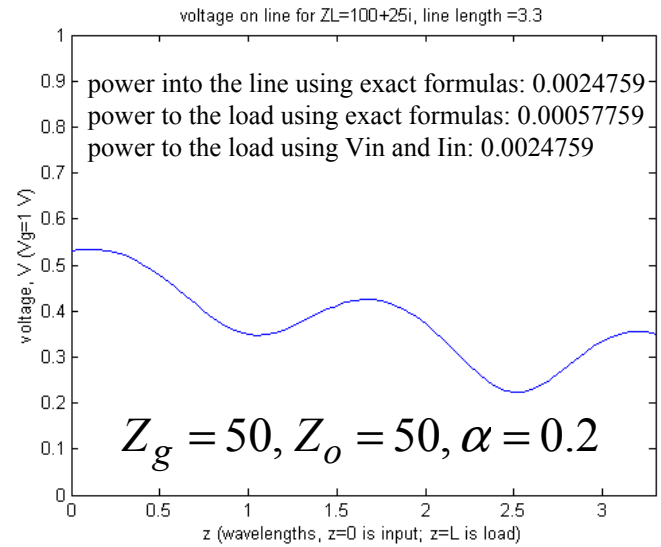
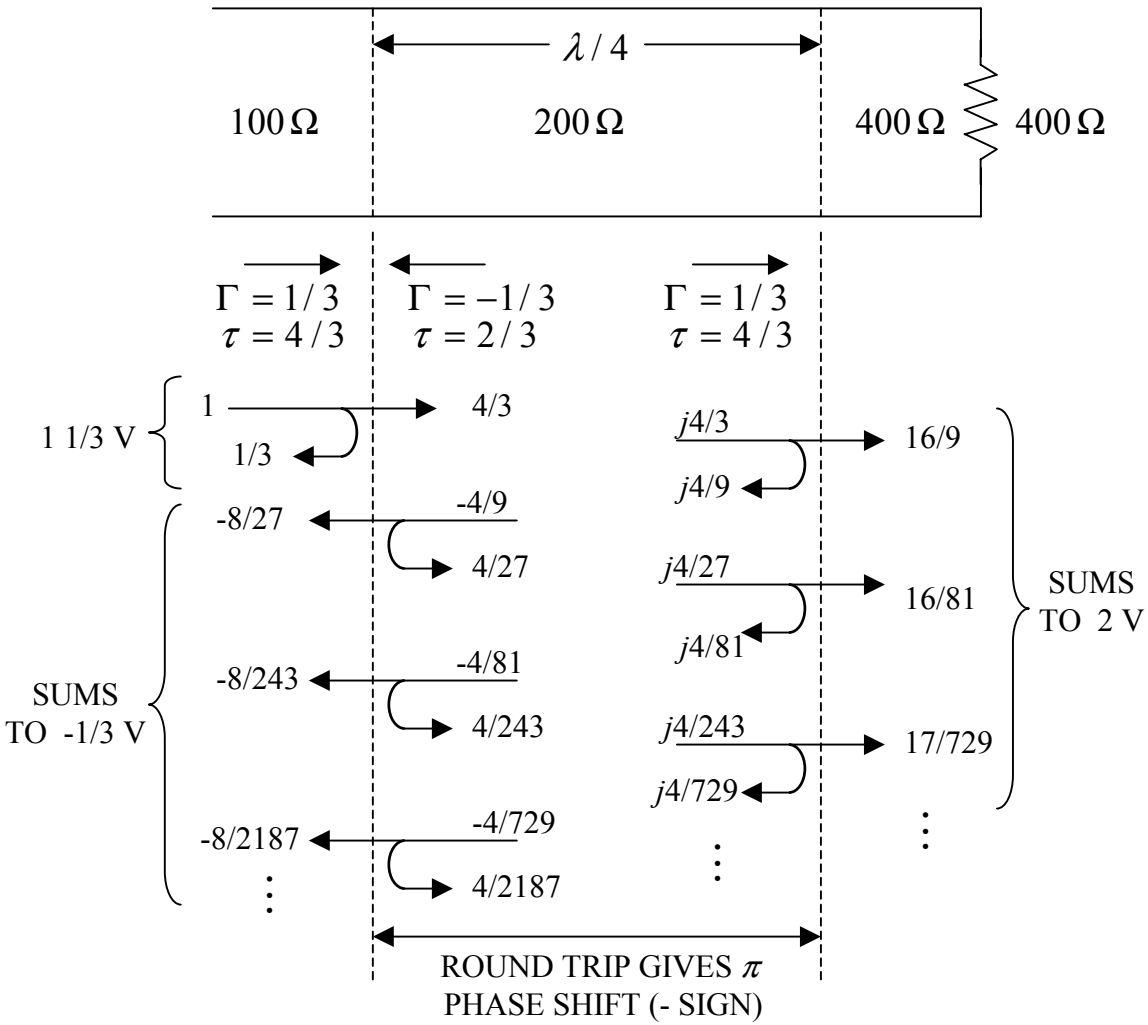
```
% compare numbers for input impedance
 clear
 zg=50;                  % generator impedanc
 z0=50;                  % characteristic impedance
 zl=25+j*25;             % load impedance
 gg=(zg-z0)/(zg+z0);     % generator reflection coefficient
 gl=(zl-z0)/(zl+z0);     % load reflection coefficient
 % use 300 MHz so wave=1m and lengths will be in wavelengths
 wave=3e8/1e8;
 L=3.3;                  % length of line in wavelengths
 b=2*pi/wave;            % beta, phase constant
 a=0.001;                % alpha, attenuation constant
 gam=a+j*b;              % gamma
 vg=1;                   % generator voltage
 vm=z0*vg/(z0+zg);
 % enter value of z (z=0 is the input; z=L is the load)
 disp(' ')
 disp(['z=0 is the input; z= ',num2str(L),' is the load'])
 %---------- calculate power into the line (z=0) -------------------
 z=0;
 zp=L-z;
 V=vm*exp(-gam*z)*(1+gl*exp(-2*gam*zp))/(1-gg*gl*exp(-2*gam*L));
 I=vg/(z0+zg)*exp(-gam*z)*(1-gl*exp(-2*gam*zp))/(1-gg*gl*exp(-2*gam*L));
 Pin=.5*real(V*conj(I));
 disp(['power into the line using exact formulas: ',num2str(Pin)])
 %---------- calculate power to the load (z=L) ---------------------
 z=L;
 zp=L-z;
 V=vm*exp(-gam*z)*(1+gl*exp(-2*gam*zp))/(1-gg*gl*exp(-2*gam*L));
 I=vg/(z0+zg)*exp(-gam*z)*(1-gl*exp(-2*gam*zp))/(1-gg*gl*exp(-2*gam*L));
 PL=.5*real(V*conj(I));
 disp(['power to the load using exact formulas: ',num2str(PL)])
 % closed form
 zin=z0*(zl+z0*tanh(gam*L))/(z0+zl*tanh(gam*L));
 Vin=vg*zin/(zin+z0);
 Iin=Vin/zin;
 Pin=.5*real(Vin*conj(Iin));
 disp(['power to the load using Vin and Iin: ',num2str(Pin)])
```

For a lossless line, the power delivered to the load is the power into the line if $Z_g = Z_o$ (no reflections at the input end of the transmission line)









Reflections on a quarter wave transformer section