MASTER OF SCIENCE IN APPLIED MATHEMATICS

APPROXIMATING THE CHROMATIC NUMBER OF AN ARBITRARY GRAPH USING A SUPERGRAPH HEURISTIC

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We color the vertices of a graph *G*, so that no two adjacent vertices have the same color. We would like to do this as cheaply as possible. An efficient coloring would be very helpful in optimization models, with applications to bin packing, examination timetable construction, and resource allocations, among others. Graph coloring with the minimum number of colors is in general an NP-complete problem. However, there are several classes of graphs for which coloring is a polynomial-time problem. One such class is the chordal graphs. This thesis deals with an experimental algorithm to approximate the chromatic number of an input graph *G*. We first find a maximal edge-induced chordal subgraph *H* of *G*. We then use a completion procedure to add edges to *H*, so that the chordality is maintained, until the missing edges from *G* are restored to create a chordal supergraph *S.* The supergraph *S* can then be colored using the greedy approach in polynomial time. The graph *G* now inherits the coloring of the supergraph *S*.

MATHEMATICAL MODELING USING MICROSOFT EXCEL

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The entry into higher mathematics begins with calculus. Rarely, however, does the calculus student recognize the full power and applications for the mathematical concepts and tools that are taught. Frank R. Giordano, Maurice D. Weir, and William P. Fox produced *A First Course in Mathematical Modeling*, a unique text designed to address this shortcoming and teach the student how to identify, formulate, and interpret the real world in mathematical terms. Mathematical modeling is the application of mathematics to explain or predict real-world behavior. Often real-world data are collected and used to verify or validate (and sometimes formulate) a hypothetical model or scenario. Inevitably, in such situations, it is desirable and necessary to have computational support available to analyze the large amounts of data. Certainly this eliminates the tedious and inefficient hand calculations necessary to validate and apply the model (assuming the calculations can even be reasonably done by hand).

The primary purpose of *Mathematical Modeling Using Microsoft Excel* is to provide instructions and examples for using the spreadsheet program Microsoft Excel to support a wide range of mathematical modeling applications. Microsoft Excel is a powerful spreadsheet program which allows the user to organize numerical data into an easy-to-follow on-screen grid of columns and rows. Our version of Excel is based on Microsoft Windows. In this text, it is not the intent to teach mathematical modeling, but rather to provide computer support for most of the modeling topics covered in *A First Course in Mathematical Modeling*. The examples given here support that text as well.

MASTER OF SCIENCE IN APPLIED MATHEMATICS

ANALYSIS OF THE NUMERICAL SOLUTION OF THE SHALLOW WATER EQUATIONS

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This thesis is concerned with the analysis of various methods for the numerical solution of the shallow water equations along with the stability of these methods. Most of the thesis is concerned with the background and formulation of the shallow water equations. The derivation of the basic equations will be given, in the primitive variable and vorticity-divergence formulation. Also the shallow water equations will be written in spherical coordinates. Two main types of methods used in approximating differential equations of this nature will be discussed. The two schemes are finite difference method (FDM) and the finite element method (FEM). After presenting the shallow water equations in several formulations, some examples will be presented. The use of the Fourier transform to find the solution of a semidiscrete analog of the shallow water equations is also demonstrated.

MARKOV RANDOM FIELD TEXTURES AND APPLICATIONS IN IMAGE PROCESSING

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In the field of image compression, transmission and reproduction, the foremost objective is to reduce the amount of information which must be transmitted. Currently the methods used to limit the amount of data which must be transmitted are compression algorithms using either lossless or lossy compression. Both of these methods start with the entire initial image and compress it using different techniques. This paper will address the use of Markov Random Field Textures in image processing. If there is a texture region in the initial image, the concept is to identify that region and match it to a suitable texture which can then be represented by a Markov random field. Then the region boundaries and the identifying parameters for the Markov texture can be transmitted in place of the initial or compressed image for that region.

PREDICTION AND GEOMETRY OF CHAOTIC TIME SERIES

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This thesis examines the topic of chaotic time series. An overview of chaos, dynamical systems, and traditional approaches to time series analysis is provided, followed by an examination of the method of state space reconstruction. State space reconstruction is a nonlinear, deterministic approach whose goal is to use the immediate past behavior of the time series to reconstruct the current state of the system. The choice of delay parameter and embedding dimension are crucial to this reconstruction. Once the state space has been properly reconstructed, one can address the issue of whether apparently random data has come from a low-dimensional chaotic (deterministic) source or from a “random” process. Specific techniques for making this determination include attractor reconstruction, estimation of fractal dimension and Lyapunov exponents, and short-term prediction.

If the time series data appears to be from a low-dimensional chaotic source, then one can predict the “continuation” of the data in the short term, exploiting the fact that chaotic systems are fairly predictable in the short term. This is the “inverse problem” of dynamical systems. In this thesis, the technique of local fitting is used to accomplish the prediction. Finally the issue of noisy data is treated, with the purpose of highlighting where further research may be beneficial.

INTERPOLATION WEIGHTS OF ALGEBRAIC MULTIGRID

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Algebraic multigrid (AMG) is a numerical method used to solve particular algebraic systems, and interest in it has risen because of its multigrid-like efficiency. Variations in methodology during the interpolation phase result in differing convergence rates. We have found that regular interpolation weight definitions are inadequate for solving certain discretized systems so an iterative approach to determine the weights will prove useful. This iterative weight definition must balance the requirement of keeping the interpolatory set of points "small" in order to reduce solver complexity while maintaining accurate interpolation to correctly represent the coarse-grid function on the fine grid. Furthermore, the weight definition process must be efficient enough to reduce setup phase costs.

We present systems involving matrices where this iterative method significantly outperforms regular AMG weight definitions. Experimental results show that the iterative weight definition does not improve the convergence rate over standard AMG when applied to M-matrices; however, the improvement becomes significant when solving certain types of complicated, non-standard algebraic equations generated by irregular operators.