MASTER OF SCIENCE
IN
APPLIED MATHEMATICS

BÉZIER CURVE FITTING

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We typically think of fitting data with an approximating curve in the linear least squares sense, where the sum of the residuals in the vertical, or *y*, direction is minimized. The problem addressed here is to fit a Bézier curve to an ordered set of data in the total least squares sense, where the sum of the residuals in both the horizontal and vertical directions is minimized. More exact: given an ordered set of *m* data points d_i, $i = 1,2,\ldots,m$ find a set of control points b_i, $i = 0,1,\ldots,n$ where *n* is the order of the Bézier curve, and a vector t of nodes, $0 \le t_1 \le t_2 \le \cdots \le t_m \le 1$ that minimize $\| B(t)P - D \|_F$. The matrix *D* contains the data points, the matrix *P* contains the control points, and the matrix *B*(t) is a Bernstein matrix. The algorithm to accomplish this is explained in detail and makes extensive use of the linear algebra representation of Bézier curves.

DoD KEY TECHNOLOGY AREA: Other (Applied Mathematics)

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