Logic Review

CS4605 – Formal Methods

Please answer the following questions on a separate sheet of paper, showing a sufficient amount of work that someone else can follow your problem-solving process. Answers are due tomorrow at the beginning of class. These questions all deal with introductory mathematical reasoning, including principles of logic, that you will need in order to understand the concepts in this course. If you have questions about this assignment, please attend the review session on Thursday at 1700 in Sp-421.

For these questions, the symbol $\wedge$ means logical AND, $\vee$ means logical OR, $\Rightarrow$ means implication ("if"), $\Leftrightarrow$ means equivalence ("if and only if", "iff"), $\oplus$ means logical XOR, and $\sim$ means logical NOT.

Truth Tables

For the questions in this section, please fill in the truth table such that the values in the truth table correspond to the result of the formula when applied to the column and row values.

Question 1

		Q	
	$P \wedge Q$	0	1
P	0		
	1		

Question 2

		Q	
	$P \vee Q$	0	1
P	0		
	1		

Question 3

		Q	
	$P \Rightarrow Q$	0	1
P	0		
	1		

Question 4

	$P \Leftrightarrow Q$	**Q**	
		0	1
P	0		
	1		

Question 5

	$P \oplus Q$	**Q**	
		0	1
P	0		
	1		

Proofs

Some of the following statements are false. Please provide a counterexample for those that are false and prove the rest. There are at least two ways to prove each true statement. All variables for questions ≤ 16 can range over the possible truth values.

Question 6

$((P \Rightarrow Q) \wedge P) \Rightarrow Q$

Question 7

$((P \wedge Q) \wedge R) \Rightarrow (P \wedge (Q \wedge R))$

Question 8

$\sim (P \vee Q) \Leftrightarrow (\sim P \wedge \sim Q)$

Question 9

$\sim (P \wedge Q) \Leftrightarrow (\sim P \vee \sim Q)$

Question 10

$(P \Rightarrow (Q \Rightarrow R)) \Leftrightarrow (P \wedge Q \Rightarrow R)$

Question 11

$(p \wedge (q \vee r)) \Leftrightarrow (p \wedge q) \vee (p \wedge r)$

Question 12

$(p \vee (q \wedge r)) \Leftrightarrow (p \vee q) \wedge (p \vee r)$

Question 13

$((p \Rightarrow q) \wedge (p \Rightarrow r)) \Leftrightarrow (p \Rightarrow (q \wedge r))$

Question 14

$((p \Rightarrow q) \Rightarrow (p \wedge q)) \Leftrightarrow ((\sim p \Rightarrow q) \wedge (q \Rightarrow p))$

Question 15

$((p \vee q) \wedge r) \Rightarrow (p \wedge (q \wedge r))$

Question 16

$(p \wedge (q \wedge r) \Rightarrow (p \vee q) \wedge r)$

Question 17

$$n(n+1)=2*\sum_{i=1}^{n} i$$

(Hint: this statement is true; you will likely want to prove it by induction on n.)

Sets

Please provide the set that is the result of the operation in each question. For these questions, $\mathbb{Z}$ is the set of integers (positive, negative and zero) and $\mathbb{N}$ is the set of natural numbers (including zero). Also, $\cap$ stands for set intersection, $\cup$ stands for set union, $\setminus$ stands for set difference, and x stands for cross product.

Question 18

$\{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$

Question 19

$\{1, 2, 3, 4\} \setminus \{3, 4, 5, 6\}$

Question 20

$\{1, 2, 3\}$ x $\{3, 4, 5\}$

Question 21

$\mathbb{Z} \setminus \mathbb{N}$

Question 22

$\mathbb{Z} \cup \mathbb{N}$

Question 23

$\mathbb{Z} \cap \mathbb{N}$