A Stochastic Program for Optimizing Military Sealift subject to Attack

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Abstract

- Describe a SP for planning the wartime, sealift deployment of military cargo.
- Question: Can such a model usefully hedge against enemy attacks on destination ports (SPODs) at uncertain times/locations?
- (Hedging involves timing and routing of ships. Recourse involves the ships waiting and re-routing.)
- A multi-stage SP says “Yes.”

1. Introduction

- USTRANSCOM plans wartime deployment of US cargo ships and cargo, from US-controlled seaports of embarkation (SPOEs) to overseas seaports of debarkation (SPODs)
- Little or no optimization used; no SP
- The purpose of this paper is

1) to develop a SP model that proactively plans for potential disruptions caused by enemy attacks on SPODs, and
2) to illustrate the potential benefit of the model in a realistic wartime deployment

- The model is designed for insight, not operational planning

1.1. Background

- Military sealift deployments are scheduled via Time-Phased Force-Deployment Data (TPFDD).
- Timeframe: 100 days
- For each cargo, it specifies time windows for loading and for intermediate and final destinations
- Developed iteratively with a ship/cargo-scheduling “model”
- Currently, all “models” are essentially rule-based systems
- TPFDDs assume no disruptions

Current planning

- JFAST schedules lift assets to execute the TPFDD
- New global command-and-control system to “make visible” all cargoes and lift assets are visible to planners
- Planners can react quickly to contingencies now, but there is no pro-active planning for them
- Biological attacks on SPODs are currently of interest (12+ countries are researching; Iraq and Russia have bio weapons; such weapons are cheap to produce)

Our approach

- Start with a deterministic MIP for scheduling cargo and ships to the TPFDD: Minimize weighted ton-days lateness for cargo
- Create a special multi-stage SP version of that to model attacks that shut down the SPODs for a period of time after an attack: Minimize expected weighted cargo lateness
- Compare “manual” and stochastic solutions for a hypothetical Middle-East deployment

The SP will exploit:

(a) Timing of ship voyages to avoid periods when attack probabilities are high (known from intelligence reports, say)

(b) Balancing shipments to alternative SPODs if only one will be attacked, although we don’t know which one

(c) Rerouting of ships to non-attacked ports

SSDM: The Stochastic Sealift Deployment Model

- Focuses on biological attack
- We assume:
  - An attack is immediately detected
  - An attacked SPOD will shut down entirely during a *decontamination period*
  - Then the port's cargo-handling capacity returns to normal following a *recovery rate*
  - The uncertain severity of the attack dictates the length of the decontamination period and recovery rate
  - A generic, Roll-On/Roll-Off cargo ship is modeled; it carries 15 ktons of cargo
  - Airlift ignored
  - At most one attack will occur

1.2. Background: SP and Uncertainty in Military Models including Deployments

- Civilian transportation under uncertainty: Ferguson and Dantzig (1956).
- Two-stage SP for scheduling monthly and daily airlift with uncertain cargo demands (Midler and Wollmer 1969).
- Simulation is the standard, e.g., The Warfighting and Logistics Technology Assessment Environment (WLTAE) links warfighting and logistics simulation models (Sinex et al. 1998).
- Brown (1999) does provide re-optimization techniques (MIP or heuristic) suitable for embedding in WLTAE.
- Deterministic and stochastic sealift deployment models at NPS: Aviles (1995), Theres (1998), Alexander (1999), Brown (1999), Loh (2000)

- **SP Background, continued**

- Deterministic airlift optimization: Killingsworth & Melody (1995), Rosenthal et al. (1997), Baker et al. (2001).
- Goggins (1995) and Niemi (2000) have SP variants (simple recourse) of Rosenthal et al. (1997) and Baker et al. (2001), resp., to incorporate aircraft reliability.
- Mulvey and Vanderbei (1995) and Mulvey and Ruszczynski (1995): The two-stage SP, "STORM," assigns aircraft to routes in the first stage; after realizing random point-to-point cargo demands, assigns cargo to aircraft.
- Powell (2001) is using simulation and dynamic programming to handle uncertainty in airlift deployments; not yet available.

SP Background, continued
Uncertainty in military in optimization models

- *Sensitivity analysis* is misused (e.g., Brooks et al. 1999; see also Wallace 2000)
- *Ad hoc analysis*: Network interdiction with uncertain interdiction effects: Whiteman (1999)
- Brooks et al. (1999) propose *exploratory analysis* for a weapons-mix problem with uncertain weapons effects and warfighting scenarios: Brute-force stochastic programming!

2. Stochastic Sealift Deployment Model (SSDM)

(1) Scenario representation of uncertain attacks
(2) a ship-movement submodel
(3) a cargo-movement submodel,
(4) linking constraints and
(5) non-anticipativity constraints (Wets 1980)
(6) attacks simulated by lost unloading and cargo-handling capacity

SSDM builds on Alexander (1999) and Loh (2000).

Scenario Representation

Variables and constraints are indexed by scenario, which encompasses

- severity of an attack
- time of an attack, and
- and location(s) of an attack,
- or indicates that no attack occurs.

Severity translates into longer or shorter decontamination periods for the SPOD and slower or faster recovery rates

Ship-Movement Submodel

- Routes a ship from SPOE to SPOD to unload, then back to an SPOE, not necessarily its origin
- An en-route ship may be re-routed in response to an attack at an SPOD
- Nominally: Fixed time to unload cargo
- If an attack occurs during unloading at an SPOD, the unloading period is extended by decon time
- Ships return to some SPOE immediately after unloading, then are reloaded or wait in inventory
- Sample variables, under scenario *a*:

vs_{edta} Number of ships starting voyages from SPOE *e* to SPOD *d* at time *t*

$vrr_{dd'ta}$ Number of ships re-routed from SPOD *d* to SPOD d' at time *t*

Cargo-Movement Submodel

- Similar to the ship-movement model but incorporates separate constraints for each commodity or “cargo package”
- Adds an echelon of variables to move cargo from SPODs to final destinations (truck/rail)
- Side constraints control movement of cargo out of the SPODs; reflect uncertain cargo capacity (attack uncertainty)
- Cargo-handling capacity goes to 0 right after an attack and during decon; returns to its pre-attack level after decon according to some recovery rate.

Sample variables, under scenario *a*:

$xrr_{cdd'ta}$ Tons of cargo *c* re-routed from SPOD *d* to SPOD d' at time *t*

xu_{cfa} Tons of unmet demand for cargo *c* at destination *f*

Linking Constraints

- Ensure that sufficient ship capacity is scheduled to carry the cargo
    - From SPOE to SPOD,
    - Being re-routed between SPODs
    - From outside an SPOD into that port
- Relaxation: Cargo can move between ships that are waiting outside an SPOD.

Sample linking constraint:

$$\sum_{c} xs_{cedta} - SCAP\ vs_{edta} \leq 0 \quad \forall\, e, d, t, a$$

Non-anticipativity constraints:

Two scenarios, *a* and *a′*, with attack times $t_a \leq t_{a'}$, are consistent if all variables associated with them are identical up to time t_a–1. Non-anticipativity constraints ensure that this is the case.

Sample non-anticipativity constraint:

$$vi_{eta'} = vi_{eta} \ \forall\, e, a, a' \in A_a^-, t \in T_a \cap T_{a'}$$

Note: For now, we use explicit non-ant. constraints and let the solver's preprocessor substitute them out

Single Attack Simplification:

Scenario tree grows quadratically instead of exponentially

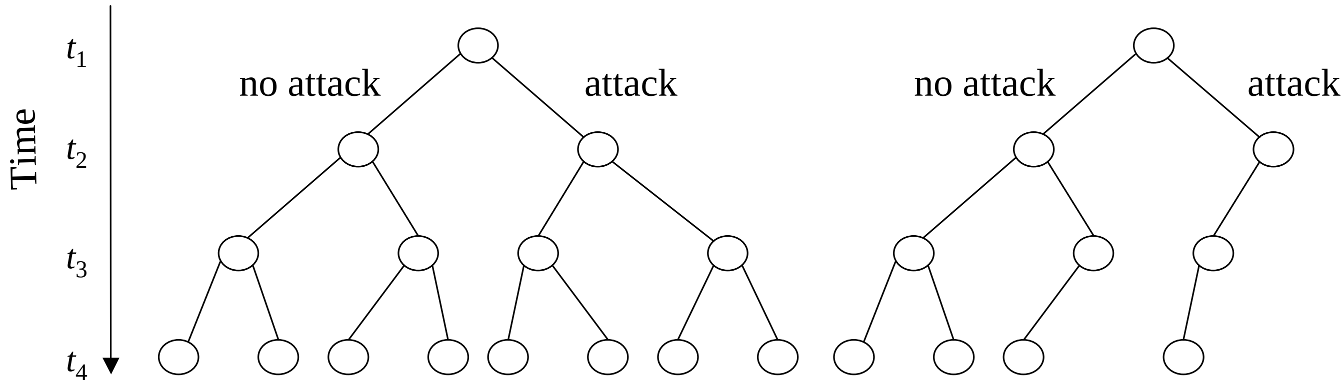

Figure 1a. A multiple-attack scenario tree

Figure 1b. A single-attack scenario tree

e.g., Infanger 1994

SSDM is more complicated because of multiple attack types

3. A Heuristic Based on SSDM

- SSDM is large; can be hard to solve
- SSDH is an heuristic based on SSDM to find good “stochastic solutions” quickly:

  1. Create a model with the no-attack scenario a_0, the “earliest-attack scenario” a_1 and a single, average “later-attack scenario” a_2
  2. Solve that model and keep the solution up to the first potential attack time
  3. Move forward in time so that the second attack is treated as the first and repeat until all potential attacks have been covered

- End up with more aggregated scenarios depending on attack locations
- A “deterministic heuristic” has led to further improvements: discussed later

4. Computational Results

- 1 GHz Pentium III computer
- 1 Gb of RAM, running under Microsoft Windows 2000
- Models are generated using GAMS
- Solved with CPLEX Version 6.5
- 5% optimality tolerance
- Solution times range from 20 minutes to several hours

4.1. Data

- Hypothetical deployment to Middle East
- 3,000 ktons of cargo in 11 packages
- 100 days in two-day time periods
- Cargo required from periods 7-45
- Four SPOEs in the US and Europe
- Two proximate SPODs (d_1 and d_2) in the Middle East
- SPOE-SPOD travel time is 6-12 periods
- 158 ships come "on line" over five wks
- Each ship transports about 15 ktons of cargo per trip
- Need > 200 SPOE-SPOD trips
- Similar to Desert Shield/Desert Storm

SPOD Data; Modeling Attacks

- Berth capacity for at most nine ships
- Normally, a ship is unloaded in two periods
- Port has 75 ktons/day of handling capacity to forward that cargo
- Single attack severity
- Cargo lateness: One ton that arrives at $t > \tau_c^{CRD}$ is penalized $(t - \tau_c^{CRD})^{1.5}$, so we are measuring kton×days$^{1.5}$

4.2. Potential "Leverage"

- First establish bounds on SSDM's potential "leverage," i.e., the potential to improve current manual planning solutions
- Simple situations:
    - An attack may occur in any period from 4 through 40, or none occurs, and
    - there are two "attack types": a single SPOD is attacked, or both SPODs are attacked simultaneously.
- Simultaneous attacks on two SPODs are called a "single attack"

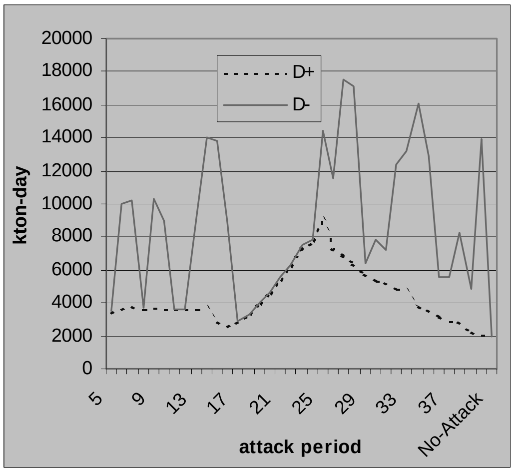

Figure 1a. Single-SPOD attack case

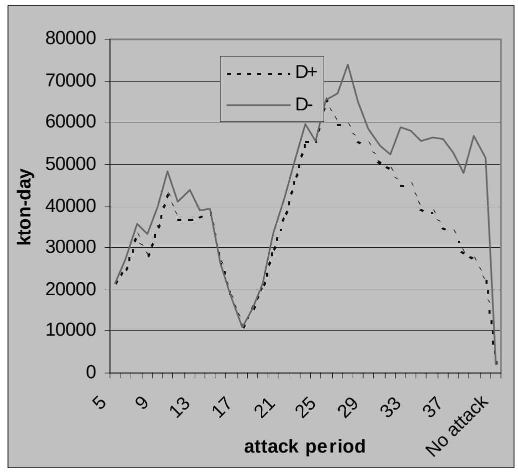

Figure 1b. Two-SPOD attack case

Figure 1. Compares “perfect knowledge” about attacks (D+) to manual “know-nothing planning” (D-).

Potential Improvement with SSDM: Bounds on EVPI

EVPI = objective of stochastic solution – objective of “perfect-information solution”

$\mathrm{EVPI} \le z^- - z^+$

$z^+ \equiv E[z_a^+]$ “wait-and-see bound”

$z^- \equiv E[z_a^-]$ know-nothing manual plan

If all scenarios have the same probability

$z^- - z^+ = 1{,}617 \text{ ktons} \times \text{days}^{1.5}$ (Figure 1a.)

and

$z^- - z^+ = 1{,}465 \text{ ktons} \times \text{days}^{1.5}$ (Figure 1b.)

The first bound on EVPI represents 33.6% of z^+, but the second is only a 7.5%.

But, 7.5% becomes larger if attacks can only occur during later time periods

4.3. More Realistic Test Cases

1. $\{d_1\},\{d_2\}$: An attack occurs at SPOD d_1 or at SPOD d_2, but not both, or no attack occurs;
2. $\{d_1\},\{d_1, d_2\}$: Mutually exclusively, an attack occurs at d_1, both SPODs are attacked simultaneously, or no attack occurs; or
3. $\{d_1\},\{d_2\},\{d_1,d_2\}$: Mutually exclusively, d_1 is attacked, d_2 is attacked, both are attacked simultaneously, or no attack occurs.

Intelligence reports tell us: The probability of no attack is $p_0 = 0.5$, but if an attack is to occur, the probability distributions are

- U1: Uniform[4,40] diffuse
- T1: Triangular[4,40,mode 22] diffuse
- U2: Uniform[4,18] early
- T2: Triangular [4,18,mode 11] early
- U3: Uniform [26,40], or late
- T3: Triangular[26,40,mode 33]. late

<table>
<tr><th rowspan="3">Attack types</th><th rowspan="3">Distribution ($|A|$)</th><th colspan="6">Problem Size</th></tr>
<tr><th colspan="3">SSDM</th><th colspan="3">DSDM</th></tr>
<tr><th>m_1</th><th>n_1</th><th>n_2</th><th>m_1</th><th>n_1</th><th>n_2</th></tr>
<tr><td>$\{d_1\}, \{d_2\}$</td><td>U2, T2, U3, T3 (31)</td><td>165,346</td><td>229,012</td><td>18,300</td><td rowspan="5">2,690</td><td rowspan="5">5,253</td><td rowspan="5">586</td></tr>
<tr><td>$\{d_1\}, \{d_2\}$</td><td>U1, T1 (75)</td><td rowspan="2">605,346</td><td rowspan="2">672,399</td><td rowspan="2">44,260</td></tr>
<tr><td>$\{d_1\},\{d_1,d_2\}$</td><td>U1, T1 (75)</td></tr>
<tr><td>$\{d_1\}, \{d_2\}, \{d_1,d_2\}$</td><td>U3, T3 (46)</td><td>480,770</td><td>475,908</td><td>27,000</td></tr>
<tr><td>$\{d_1\}, \{d_2\}, \{d_1,d_2\}$</td><td>U1, T1(112)</td><td>906,674</td><td>>1M</td><td>85,720</td></tr>
</table>

Table 1. Problem definitions/sizes for the stochastic sealift deployment model SSDM and its deterministic restriction DSDM.

m = constraints

n_1 = continuous variables

n_2 = binary variables

Sizes for SSDM are “raw”

Attack types	Distri. (|A|)	D^+	SSDM	SSDH	DH^-	D′	SSDM′	SSDH′
$\{d_1\}$, $\{d_2\}$	U2 (31)	2,746	4,455	5,327	4,479	1,974	1,975	3,169
	T2 (31)	2,727	4,584	5,270	4,641		2,016	2,920
	U3 (31)	3,062	4,534	5,284	6,140		2,011	2,307
	T3 (31)	3,044	4,466	5,208	6,573		1,996	2,257
	U1 (75)	3,150	4,676	5,449	5,583		2107	2,336
	T1 (75)	3,486	4,900	5,597	5,603		2115	2,436

Table 2a. One SPOD or the other, not both. Compare general, early and late attacks

- Early attacks: improvement small
- Late or diffuse attacks, SSDM reduces expected disruption by $700-2{,}100 \text{ ktons}\times\text{days}^{1.5}$
- SSDM robust with no attack
- SSDH shows promise (see U2,U3), but a meta-heuristic is best

Attack types	Distribution (|A|)	D^+	SSDM	SSDH	DH^-	D'	SSDM′	SSDH′
$\{d_1\}$, $\{d_1,d_2\}$	U1 (75)	3,325	4,340	4,723	5,457	1,974	2,055	2,258
	T1 (75)	3,485	4,385	5,645	5,562		2,009	2,541
$\{d_1\}$, $\{d_2\}$, $\{d_1,d_2\}$	U3 (75)	9,046	10,425	11,181	13,594		2,154	2,216
	T3 (75)	9,134	10,242	10,399	13,538		2,130	2,030
	U1 (112)	8,884	9,749*	11,273	11,958		-	2,214
	T1 (112)	9,449	10,286*	11,540	11,926		-	2,115

Table 2b. More complicated attack types

- 1 and 2: Diffuse attacks, SSDH fair
- 3 and 4: Late attacks, SSDH good
  Improves $3{,}000 \text{ ktons} \times \text{days}^{1.5}$
- SSDM robust OK with no attack
- But, is there a smarter “dumb heuristic” for later attacks?

4.4. An Improved, Special-Case "Deterministic Heuristic"

- When attacks can only occur late, try pushing cargo early
- Will penalize late deliveries as before, but encourage early ones:

$$-\beta(\tau_c^{CRD} - t)^\alpha \text{ for } t < \tau_c^{CRD}$$

β	DH⁻(β)	D⁻	SSDM	SSDH	SSDH′(β)
0.00	6,140	6,140	4,534	5,284	5,388
0.05	4,982				4,851
0.10	4,899				4,770
0.15	5,105				4,756
0.25	5,126				5,084
0.50	6,105				5,337
1.00	8,020				7,144

Table 3. A smarter deterministic heuristic, but a smarter stochastic one, too.
Test set: {{d_1},{d_2}}, U3

4.5. Different Attack Severities

Not finished—well, not started.

Probably won’t finish for Norge.

4.6. Summary of Computational Results

- The manual plan may be notably worse than the stochastic plan (SSDM); differences may exceed $3,000 \text{ ktons} \times \text{days}^{1.5}$ (about 30%)
- If an attack does not occur, the stochastic solution is still robust: The extra disruption is typically less than 2%
- Heuristics are promising for larger problems that SSDM may not be able to handle

5. Conclusions

- Proactive planning for potential biological attacks during a military sealift deployment can yield substantially better on-time cargo deliveries
- There is little penalty involved if an attack does not occur
- So, quick reactions to contingencies is important, but pre-planning for them is too.
- Currently building a GUI to allow planners to experiment with SSDM
- Improvements and extensions
  - Other wartime scenarios
  - More ship types
  - Multiple attacks

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