ECON 2106 - Microeconomics
Chapter 22 - Cost and Output Determination

1. Production Functions - In General

Production functions are used in economics to denote the relationship between inputs (capital, labor) and outputs (goods, works, and services).

Mathematically, we can specify a production function as:

$Q = f(k, l, r, ...)$

Where Q is output, K is capital, L is labor, R is land, and so forth. Thus, for example, if a table required the input of 5 hours of labor and 1 hour machine time and 10 lbs of wood, the production function for a table would be:

1 table = f (1 hour machine time, 5 hours labor, 10 lbs of wood)

2. Average and Marginal Physical Products of Labor

Note that in the previous expression, we defined output (q) as a function of land, labor, capital, and entrepreneurship. We will now focus on the relationship between output and a combination of two inputs: capital and labor.

Let's specify a production function: $q = f(k, l)$ where output is dependent upon inputs of capital and labor. Recall that the law of diminishing returns also applies to production, so that we know that when, past some point, successive units of input are added, the marginal increase in output declines.

Assume that we hold one of the input resources constant, that is, we hold either capital or labor constant, then we can state the law of diminishing marginal returns as:

The law of diminishing marginal returns occurs when equal increases of variable resources are successively added to some fixed resource; marginal physical products eventually decline.

We can examine diminishing marginal returns by looking at the relationship between labor, output, average physical product, and marginal physical product.

Definition:

The Average Physical Product of Labor (APP_L) is equal to the total output (Q) divided by total labor input (L), or

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$$APP_L = Q / L$$

The Marginal Physical Product of Labor (MPP_L) is equal to the change in total output divided by the change in total labor or:

$$MPP_L = \Delta Q / \Delta L$$

Average physical product is merely the <u>average output per unit of labor</u>. In the table below, labor could be denoted in number of workers per hour, per shift, or per day. APP_L is merely total output divided by total labor or Q / L

However, as we have noted previously in out examinations of the relationship between consumption and utility, we are not especially interested in average measures but in marginal changes in the consumption of a product. In the production sense, we are interested in the marginal output per unit of input.

Thus, with respect to labor, we are interested in the <u>marginal physical product of labor</u> where the marginal physical product of labor is equal to the change in total quantity divided by the change in labor. These measures are illustrated in the table below.

Labor	Output	Average Physical Product	Marginal Physical Product
0	0	0	0
1	5	5	5
2	12	6	7
3	20	6.66	8
4	29	7.25	9
5	40	8	11
6	50	8.3	10
7	58	8.25	8
8	64	8	6
9	68	7.56	4
10	70	7	2

11	66	6	-4

ILLUSTRATE GRAPH OF TOTAL PRODUCT (Q), APP, and MPP

Note that diminishing marginal returns plays a part in the production process. In the short-run, recall, at least one of the inputs is fixed, so past some point, when additional units of input are added, the marginal contribution of those additional units starts to decline.

In the above example, diminishing marginal returns occurs after the addition of the third unit of labor.

Also, note the relationship between the average and the margin. When the margin is greater than value than the average, the average is increasing, however, when the margin is less than the average in value, the average must be decreasing

3. Short-Run Costs

We can generally classify production costs into two categories: **fixed costs** and **variable costs**.

Fixed costs are those costs that a firm would incur invariant of the level of production, that is, the firm would have to pay these costs no matter if it produced nothing or if it was producing at full capacity. Thus the term fixed costs. Example of fixed costs include rent, interest on loans, and other payments that are not related to production.

Variable costs, on the other hand, are costs directly associated with production. As the level of output increases, variable costs increase. Examples of variable costs include labor, depreciation, utilities, raw materials, and so on.

Total Costs are equal to **Total Fixed Costs** plus **Total Variable Costs** or:

TC = TFC + TVC

From this, we can calculate

Average Fixed Costs (TFC/Q)
Average Variable Costs (TVC/Q)
Average Total Costs (TC/Q)
Marginal Costs (Change in TC / Change in Q)

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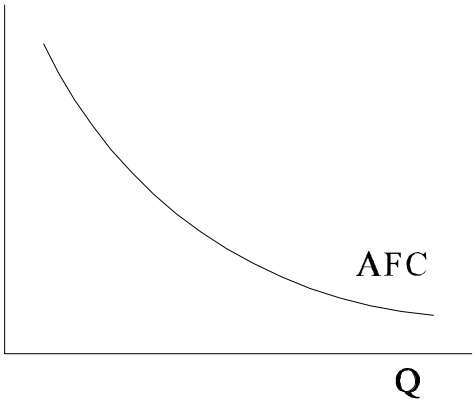

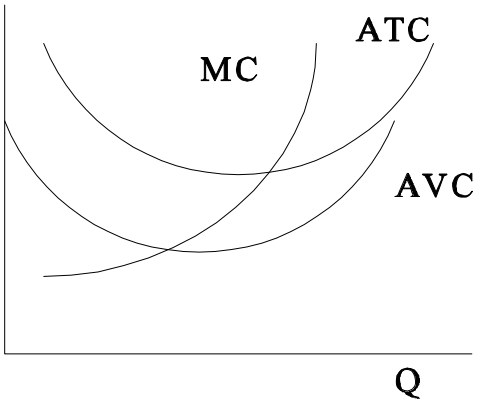

Given that TFC is a fixed sum, AFC declines through the range of production.

With respect to AVC and MC, MC lies below AVC for the initial range of production, thus AVC declines due to economies of scale, that is, as you increase production from 0, your average costs decline as you become more efficient. Eventually, diminishing marginal returns sets in and MC starts to increase.

Note that as MC is increase, MC intercepts AVC and ATC at its minimum point. This is a mathematical certainty as when the marginal value is equal the average value, the average value must be at its minimum. To the left of the minimum, MC < AVC, so AVC is declining, and to the right of the minimum, MC > AVC, so AVC is increasing.

Total Output (Q)	**Total Fixed Costs (TFC)**	**Total Variable Costs (TVC)**	**Total Costs (TC)**	**Average Fixed Costs (AFC)**	**Average Variable Costs (AVC)**	**Average Total Costs (ATC)**	**Marginal Costs (MC)**
0	20	0	20	----	----	----	----
1	20	10	30	20	10	30	10
2	20	18	38	10	9	19	8
3	20	25	45	6.66	8.33	15	7
4	20	30	50	5	7.5	12.5	5
5	20	40	60	4	8	12	10
6	20	60	80	3.33	10	13.33	20

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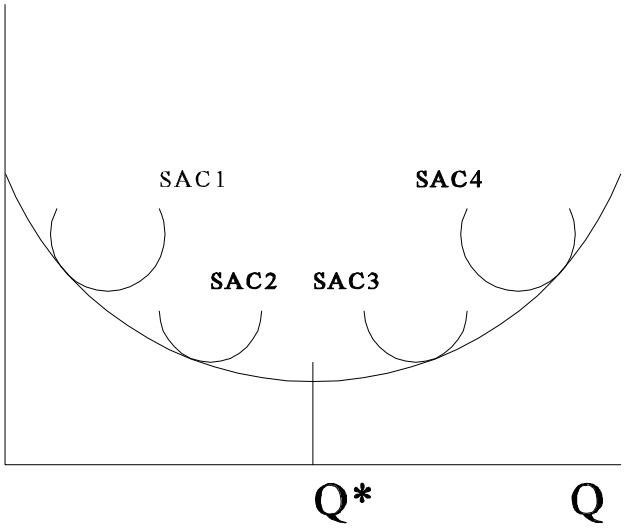
4. Long-Run Costs

We now turn our attention to costs and production in the long-run. Recall, that in the long-run, all inputs in the production process are variable. Thus, in the long-run, firms can increase or decrease capacity as necessity to adjust to changes in market demand.

The decision horizon of the firm is such that it can choose from a series of plant sizes, each with an efficient production output. The firm, with cost minimization in view, will typically choose the plant size so that Average Total Costs are minimized. Thus, a larger plant might have a lower minimum ATC, but if demand is such that a smaller capacity is warranted, the efficiency gains of the larger plant will not be realized and the firm will actually incur greater average costs.



We can then take all the short-run average total cost curves and develop the long-run average total cost curve. The long-run ATC represents the lease unit costs of producing any given rate of output. Note that the LRATC is not tangent to each individual SRATC curve's minimum point. This is true only at the minimum point of the LRATC.

Note that to the left of Q*, the firm is experiencing **economies of scale**, that is, an increase in scale and production leads to a fall in unit costs. To the right of Q*, the firm is experiencing **diseconomies of scale**, that is, an increase in scale and production leads to an increase in unit costs. When the LRATC is at its minimum point, the firm is experiencing **constant returns to scale**, such that an increase in scale and production does not result in a change in unit costs.

Finally, we can explore the topic of **minimum efficient scale**. At the output rate when economies of scale end and constant returns to scale begin, the minimum efficient scale of the firm is encounters. The minimum efficient scale will always be the lowest rate of output at which LRATC is at its minimum, or the beginning of the constant returns to scale.

Note that when MES is small relative to industry demand, the degree of competition in that industry is likely to be high since there is room for a large number of small efficient firms. Conversely, when the MES is large relative to industry demand, the degree of competition is likely to be smaller, because there is room only for a small number of large firms.