	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\hat{r}$	$\cos\phi$	$\sin\phi$	0
$\hat{\phi}$	$-\sin\phi$	$\cos\phi$	0
$\hat{z}$	0	0	1

Rectangular and cylindrical

	$\hat{x}$	$\hat{y}$	$\hat{z}$
$\hat{R}$	$\sin\theta\cos\phi$	$\sin\theta\sin\phi$	$\cos\theta$
$\hat{\theta}$	$\cos\theta\cos\phi$	$\cos\theta\sin\phi$	$-\sin\theta$
$\hat{\phi}$	$-\sin\phi$	$\cos\phi$	0

Rectangular and spherical

	$\hat{r}$	$\hat{\phi}$	$\hat{z}$
$\hat{R}$	$\sin\theta$	0	$\cos\theta$
$\hat{\theta}$	$\cos\theta$	0	$-\sin\theta$
$\hat{\phi}$	0	1	0

Cylindrical and spherical

Example: from top table, reading across,

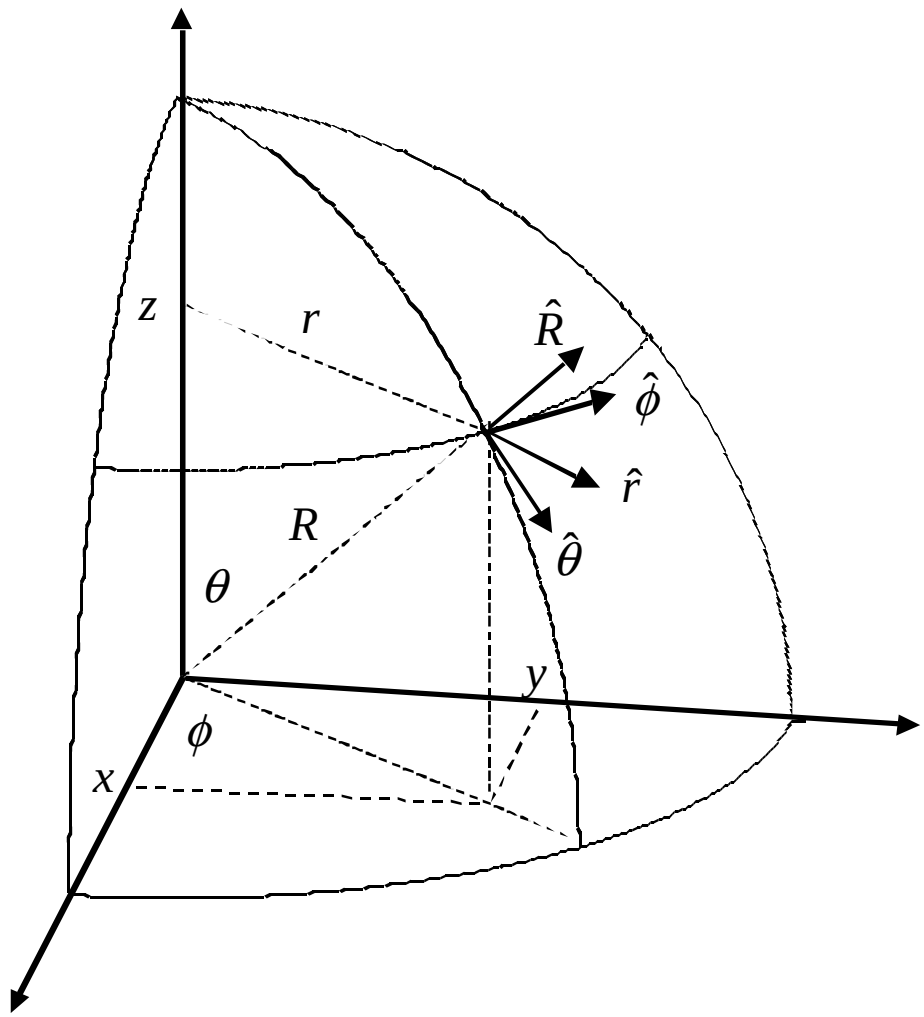
$$\hat{r} = \hat{x}\cos\phi + \hat{y}\sin\phi$$

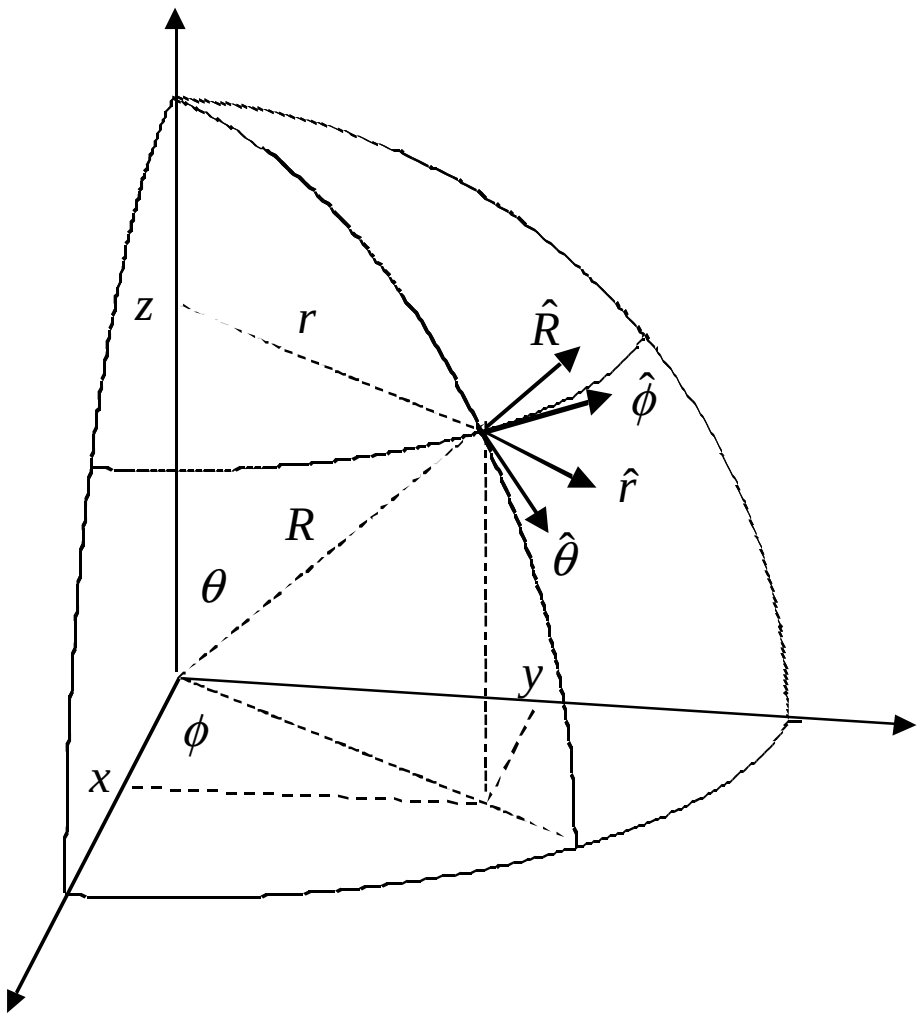
and reading down,

$$\hat{x} = \hat{r}\cos\phi - \hat{\phi}\sin\phi$$

The tables also be used to transform vectors. For example, given $\vec{A} = A_x\hat{x} + A_y\hat{y} + A_z\hat{z}$ in cartesian coordinates, the vector can be expressed in cylindrical coordinates

$$A_r = A_x\cos\phi + A_y\sin\phi + A_z \cdot 0$$


z
r
$\hat{R}$
$\hat{\phi}$
$\hat{r}$
R
$\hat{\theta}$
θ
y
ϕ
x



$$R = \sqrt{x^2 + y^2 + z^2} = \sqrt{r^2 + z^2}$$

$$\phi = \tan^{-1}\left(\frac{y}{x}\right)$$

$$\theta = \tan^{-1}\left(\frac{r}{z}\right)$$